$i^2=-1$

Nombres Complexes

$(1+i)^2=2i$

$(1-i)^2=-2i$

* Forme algebrique

- $z=a+ib$ avec $a=Re(z)$ et $b=Im(z)$

Conjugué d'un nombre Complexe.

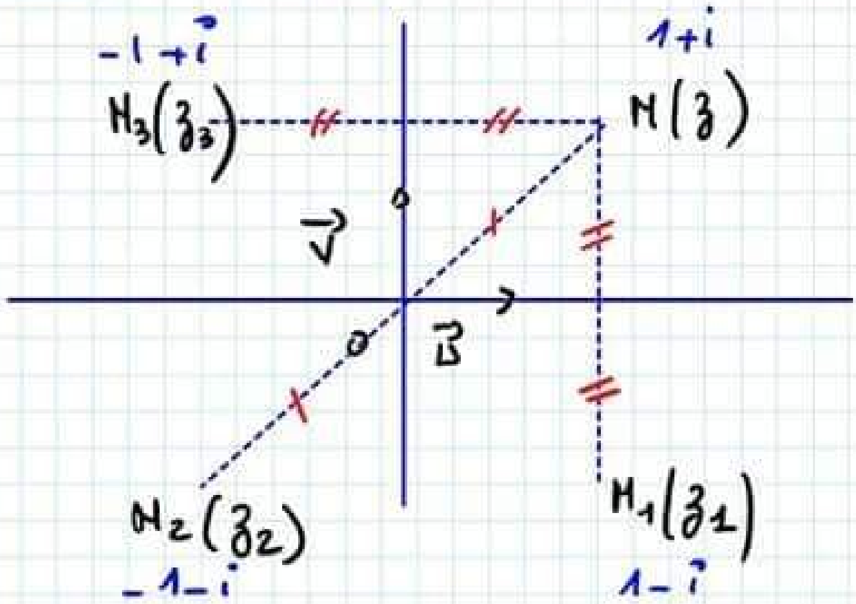
- $z=1+3i \longrightarrow \overline{z}=1-3i$
- $z=-5i \longrightarrow \overline{z}=5i$
- $z_1=i-z \longrightarrow \overline{z_1}=-i-\overline{z}$
- $z_2=1-iz^2 \longrightarrow \overline{z_2}=1+i\overline{z}^2$
- $\boxed{z+\overline{z}=2Re(z)=2x}$; $\boxed{z-\overline{z}=2i\,Im(z)=2iy}$

$$\boxed{z\overline{z}=Re^2(z)+Im^2(z)=x^2+y^2=|z|^2}$$

- $\overline{\overline{z}}=z$; $\overline{z^2}=(\overline{z})^2$

Repère Complexe.

- $M_1 = S_{(O,\vec{u})}(M)$
  $\Rightarrow \boxed{z_1 = \overline{z}}$
- $M_2 = S_O(M)$
  $\Rightarrow \boxed{z_2 = -z}$
- $M_3 = S_{(O,\vec{v})}(M)$
  $\Rightarrow \boxed{z_3 = -\overline{z}}$



- $z_A = 2+3i \longrightarrow A(2,3)$
- $z_M = x+iy \longrightarrow M(x,y)$
- $\boxed{z_{\overrightarrow{AB}} = z_B - z_A}$; $\boxed{z_M = z_{\overrightarrow{OM}}}$.
- ABCD est un plg (A, B et C non alignés) $\Leftrightarrow$ $z_{\overrightarrow{AB}} = z_{\overrightarrow{DC}}$.
  ou
  $A * C = B * D$
  $\Updownarrow$
  $\dfrac{z_A + z_C}{2} = \dfrac{z_B + z_D}{2}$

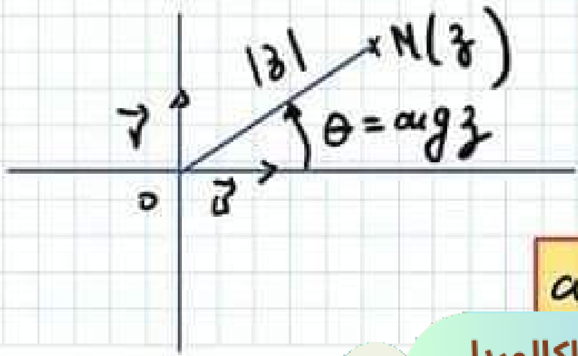
- ABCD est un losange $\Leftrightarrow$ plg + $\boxed{AB = AD}$
- ABCD est un rectangle $\Leftrightarrow$ plg $\boxed{+ \dfrac{z_{\vec{AB}}}{z_{\vec{AB}}} \in i\mathbb{R}}$
- ABCD est un carré $\Leftrightarrow$ plg $\boxed{+ AB = AD}$ $\boxed{+ \dfrac{z_{\vec{AB}}}{z_{\vec{AB}}} \in i\mathbb{R}}$
- ABCD est un trapèze

$$\Rightarrow \frac{z_{\vec{AB}}}{z_{\vec{DC}}} \in \mathbb{R}$$

Forme trigo – Forme expo

- $z = \underbrace{|z|(\cos\theta + i\sin\theta)}_{\text{F.Trigo}} = \underbrace{|z|\,e^{i\theta}}_{\text{F.expo}}$ $\qquad z = x+iy$

$\boxed{|z| = \sqrt{x^2+y^2} \ : \text{ module de } z}$; $\boxed{\theta = \arg z \ [2\pi]}$



$\boxed{|z| = OM}$

$\boxed{(\vec{u}, \vec{OM}) \equiv \arg z \ [2\pi]}$

$\boxed{\arg(z_{\vec{AB}}) \equiv (\vec{u}, \vec{AB}) \ [2\pi]}$

$$\arg\left(\frac{z_{\vec{AB}}}{z_{\vec{CD}}}\right) = \left(\vec{CD}, \vec{AB}\right)[2\pi].$$

module

- $|z \times z'| = |z| \times |z'|$
- $\left|\frac{z'}{z}\right| = \frac{|z'|}{|z|}$
- $|z^n| = |z|^n.$
- $\boxed{z\bar{z} = |z|^2 \boxed{\neq} z^2}$

argument.

- $\arg(z \times z') \equiv \arg z + \arg z' [2\pi]$
- $\arg\left(\frac{z'}{z}\right) \equiv \arg z' - \arg z [2\pi]$
- $\arg(z^n) \equiv n \arg z [2\pi]$
- $\arg \bar{z} \equiv -\arg z [2\pi]$

$\boxed{\arg(i\alpha) \equiv \frac{\pi}{2}[2\pi] \quad \alpha \in \mathbb{R}_+^*}$; $\boxed{\arg(i\alpha) \equiv -\frac{\pi}{2}[2\pi] \quad \alpha \in \mathbb{R}_-^*}$

$\boxed{\arg(\alpha) \equiv 0 [2\pi] \quad \alpha \in \mathbb{R}_+^*}$; $\boxed{\arg(\alpha) \equiv \pi [2\pi] \quad \alpha \in \mathbb{R}_-^*}$

$\arg(2017) \equiv 0 [2\pi]$; $\arg(-2016) \equiv \pi [2\pi]$

$\arg(5i) \equiv \frac{\pi}{2} [2\pi]$; $\arg(-2i) \equiv -\frac{\pi}{2} [2\pi]$

- $\arg(1-i\sqrt{3}) \equiv \theta\ [2\pi]$

$$|1-i\sqrt{3}| = \sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$$

$$\begin{cases} \cos\theta = \dfrac{\mathrm{Re}(1-i\sqrt{3})}{|1-i\sqrt{3}|} = \dfrac{1}{2} \\ \sin\theta = \dfrac{\mathrm{Im}(1-i\sqrt{3})}{|1-i\sqrt{3}|} = \dfrac{-\sqrt{3}}{2} \end{cases} \qquad \theta = -\frac{\pi}{3}.$$

Donc $\boxed{\arg(1-i\sqrt{3}) \equiv -\frac{\pi}{3}\ [2\pi].}$

$$\boxed{\underbrace{1-i\sqrt{3}}_{\text{F. Alg}} = \underbrace{2\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)}_{\text{F. Trig}} = \underbrace{2e^{-i\frac{\pi}{3}}}_{\text{F expo}}.}$$

- $e^{i\theta} = \cos\theta + i\sin\theta$
- $\boxed{|e^{i\theta}| = 1}$, $\boxed{\arg e^{i\theta} \equiv \theta\ [2\pi]}$
- $\arg(-e^{i\theta}) \equiv \arg(-1) + \arg(e^{i\theta})\ [2\pi]$

$$\equiv \pi + \theta\ [2\pi].$$

- $\arg(2e^{i\theta/2}) \equiv \theta/2\ [2\pi].$
- $\arg(2i\,e^{i\theta}) \equiv \frac{\pi}{2} + \theta\ [2\pi]$
- $\arg(-2i\,e^{i\theta}) \equiv -\frac{\pi}{2} + \theta\ [2\pi]$

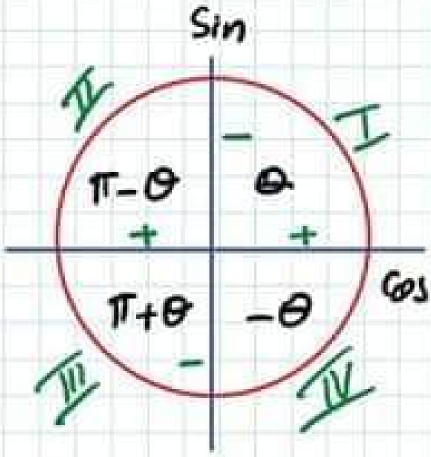
$\square$ $\arg(2i + e^{i\theta}) = \,?\ [2\pi]$.

I $\square$ $\cos\theta + i\sin\theta = e^{i\theta}$

IV $\square$ $\cos\theta - i\sin\theta = e^{-i\theta}$

II $\square$ $-\cos\theta + i\sin\theta = e^{i(\pi-\theta)}$

III $\square$ $-\cos\theta - i\sin\theta = e^{i(\pi+\theta)}$



$\square$ $\arg(-\cos\theta + i\sin\theta) = \pi - \theta\ [2\pi]$

$\square$ $\sin\theta + i\cos\theta = i(\cos\theta - i\sin\theta)$

$$= e^{i\frac{\pi}{2}} \cdot e^{-i\theta} = e^{i\left(\frac{\pi}{2}-\theta\right)}$$

$e^{i\frac{\pi}{2}} = i$

$e^{-i\frac{\pi}{2}} = -i$

$e^{i0} = 1$

$e^{i\pi} = -1$

$e^{i\theta} \cdot e^{i\theta'} = e^{i(\theta+\theta')}$

$\dfrac{e^{i\theta'}}{e^{i\theta}} = e^{i(\theta'-\theta)}$

$(e^{i\theta})^n = e^{in\theta}$

$2e^{i\frac{\pi}{3}} =$

SHIFT (−)

Formule d'Euler.

- $2\cos\theta = e^{i\theta} + e^{-i\theta}$
- $2i\sin\theta = e^{i\theta} - e^{-i\theta}$
- $-2i\sin\theta = e^{-i\theta} - e^{i\theta}$

- $1 + e^{i\theta} = e^{i\frac{\theta}{2}}\left(e^{-i\frac{\theta}{2}} + e^{i\frac{\theta}{2}}\right) = 2\cos\frac{\theta}{2}\, e^{i\frac{\theta}{2}}$.
- $1 - e^{i\theta} = e^{i\frac{\theta}{2}}\left(e^{-i\frac{\theta}{2}} - e^{i\frac{\theta}{2}}\right) = -2i\sin\frac{\theta}{2}\, e^{i\frac{\theta}{2}}$

Equation du 2ème degré

$$E : az^2 + bz + c = 0 \quad (a \neq 0)$$

- Si a, b et $c \in \mathbb{R}$ et $\Delta < 0$ alors E admet 2 solutions conjuguées
- Si E admet 2 solutions distincts z' et z'' on a :

- $\boxed{z' + z'' = -\frac{b}{a}}$ $\longrightarrow$ $\boxed{\text{si } b = 0}$ E admet 2 solutions opposées.
- $\boxed{z' \times z'' = \frac{c}{a}}$ $\longrightarrow$ si $a = c$ E admet 2 solutions inverses.

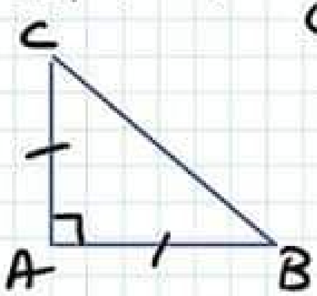
$\Delta = b^2 - 4ac = \delta^2$ $\boxed{\text{OU}}$ $\Delta' = b'^2 - ac = \delta'^2$ $\boxed{b' = \frac{b}{2}}$

$z' = \frac{-b-\delta}{2a}$ $\qquad$ $z' = \frac{-b'-\delta'}{a}$

$z'' = \frac{-b+\delta}{2a}$ $\qquad$ $z'' = \frac{-b'+\delta'}{a}$.

Remarque importante dans la pratique.

Mq ABC est un triangle rectangle isocèle en A



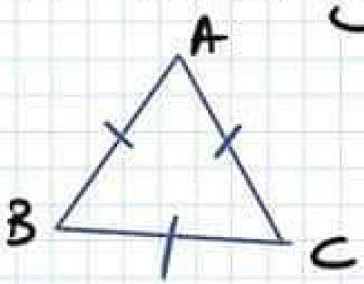
$$\boxed{\frac{z_{\overrightarrow{AB}}}{z_{\overrightarrow{AC}}} = \frac{z_B - z_A}{z_C - z_A} = \pm i.}$$

En effet. $\frac{z_{\overrightarrow{AB}}}{z_{\overrightarrow{AC}}} \in i\mathbb{R}$ donc $\overrightarrow{AB} \perp \overrightarrow{AC}$ (1)

$$\left|\frac{z_{\overrightarrow{AB}}}{z_{\overrightarrow{AC}}}\right| = |\pm i| \Leftrightarrow \frac{AB}{AC} = 1 \Leftrightarrow AB = AC \quad (2)$$

$\boxed{(1) \text{ et } (2) \Longrightarrow ABC \text{ est rectangle isocèle en } A}$

Mq ABC est un triangle équilateal



$$\frac{z_{\vec{AB}}}{z_{\vec{AC}}} = \frac{z_B - z_A}{z_C - z_A} = e^{\pm i\frac{\pi}{3}}$$

En effet : $\left|\frac{z_{\vec{AB}}}{z_{\vec{AC}}}\right| = 1 \Rightarrow \frac{AB}{AC} = 1$

$\Rightarrow \boxed{AB = AC}$ (1)

$$\arg\left(\frac{z_{\vec{AB}}}{z_{\vec{AC}}}\right) \equiv \arg(z_{\vec{AB}}) - \arg(z_{\vec{AC}}) [2\pi]$$

$$\equiv (\widehat{\vec{u}, \vec{AB}}) - (\widehat{\vec{u}, \vec{AC}}) [2\pi].$$

$$\equiv (\vec{AC}, \vec{u}) + (\widehat{\vec{u}, \vec{AB}}) [2\pi]$$

$$\equiv (\widehat{\vec{AC}, \vec{AB}}) [2\pi].$$

$$\equiv \boxed{\pm \frac{\pi}{3} [2\pi].}$$ (2)

(1) et (2) $\Rightarrow$ ABC est équilateal

$$E : az^3 + bz^2 + cz + d = 0 \quad (a \neq 0)$$

Si z_0 solution de E . alors

$$az^3 + bz^2 + cz + d = (z - z_0)(az^2 + \alpha z + \beta)$$

α et β ā determiner.

$$E : (z - z_0)(az^2 + \alpha z + \beta) = 0.$$

Sig $z - z_0 = 0$ ou $az^2 + \alpha z + \beta = 0$

$$z^n = re^{i\theta}.$$

$$\Leftrightarrow z_k = \sqrt[n]{r}\, e^{i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)}$$

avec $0 \leq k \leq n-1$